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Exchange Rate Determination and Equity Prices: Evidence from the U.K.

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Abstract

This paper develops a model of optimal choice over an array of different assets, including domestic and foreign bonds, domestic and foreign equities, and domestic and foreign real money balances, with a view to examine whether stock markets have an effect on the exchange rate in the long-run. The model is tested using data from the UK and the USA. Evidence suggests that the UK stock market has a significant effect on the value of the pound's sterling nominal effective exchange rate in the long-run over the period 1980 to 2011.

Key Words: Exchange rates, stock prices, Co-integration

JEL Classifications: F31, E44, F21, G11, G15

1. Introduction

Following the increasing flows of capital between international financial markets, in recent years, there has been considerable effort to investigate the effect that stock markets have on the exchange rates. This relationship has gained substantial interest following the deregulation of the financial markets and the abolition of capital controls especially during the 1980s. Over the following decades there have been many occasions when national stock market movements appear to have led the respective exchange rates. Evidence on such an effect was, for example, particularly apparent during the East Asian Financial crisis where causality was reported mainly from stock prices to the exchange rates (Granger et al., 2000).

The aim of this paper is to investigate the statistical association between exchange rates and stock market prices (allowing for other relevant factors) with a view to examining whether stock markets have an effect on the exchange rate in the UK over the long-run.[1] The main vehicle adopted in the literature in order to examine this relationship has been the monetary approach to the exchange rate (M.A.ER). The conventional M.A.ER model has recently been augmented in order to incorporate explicitly stock price effects on the grounds that stock prices can have a direct effect on the demand for money balances. Such models, based on the inclusion of asset market effects in the money demand equation, have recently been applied to the UK economy as an attempt to further explore the stock price effect on the exchange rate.

As distinct from the augmented M.A.ER model, this paper contributes towards the portfolio balance approach by constructing a two country model with optimizing agents where wealth is assumed to be allocated optimally in an asset choice set that includes explicitly investment in an array of assets including domestic and foreign bonds, domestic and foreign equities and domestic and foreign real money balances. To date such intertemporal optimization models, incorporating the above array of assets, have been neglected in studies of the long-run

[1] The statistical analysis abstracts from short term fluctuations around the relevant trends.

relationship between stock market prices and nominal exchange rates. The model specification introduced here allows the construction of explicit equations for both domestic and foreign real money balances, which can further be utilized in order to generate a relationship that reflects the stock price effect (among other variables) on the nominal exchange rate in the long-run. For the sake of brevity the model is applied to the UK as characteristic of a small open economy, although clearly it could be applied more widely. Moreover, comparisons will be made with the predictions of the augmented M.A.ER model, an approach for which the microfoundations are at best implicit.

The rest of this paper is organised as follows: Section 2 briefly analyses the conventional M.A.ER model with reference to relevant literature on the long-run relationship between stock prices and exchange rates. Section 3 presents the constructed intertemporal optimization model, as a contribution of understanding the stock market effect on the nominal exchange rate in the long-run. Section 4 analyses an empirical methodology for examining the predicted relationship. Section 5 discusses the data employed and presents the long-run results of the constructed economic model. A comparison of the predictions of the model with those coming from the augmented M.A.ER is also attempted. Finally, Section 6 concludes.

2. The Monetary Approach of the Nominal Exchange Rate and related literature

The formation of the flexible price monetary model for the determination of the exchange rates, as presented in the literature, stems from a variant of the log-linear Cagan (1956) model applied into conditions of moderate inflation. Real output is assumed to be exogenous and the demand for domestic real money balances is characterized by the following equations[2]:

$$m_t - p_t = -\eta i_{t+1} + \phi y_t \quad (1)$$

$$m_t^* - p_t^* = -\eta i_{t+1}^* + \phi y_t^* \quad (2)$$

[2] The superscript (*) denotes a foreign economic variable.

where m_t is the log of nominal money supply at date t, p_t is the log of the price level, defined as the price of a specified basket of consumption goods in terms of money, $\imath_{t+1} \equiv \log(1 + i_{t+1})$ with i_{t+1} as the date t interest rate on bonds denominated in home currency, y_t the log of real output at date t and η, ϕ are parameters.

Assuming purchasing power parity (PPP), i.e. $p_t = e_t + p_t^*$, and uncovered interest rate parity, i.e. $\imath_{t+1} = \imath_{t+1}^* + E_t e_{t+1} - e_t$, both hold, where e_t is the log of the nominal exchange rate defined as the amount of domestic currency per unit of foreign currency, and $E_t(\cdot)$ the mathematical conditional expectation, Equations 1 and 2 imply that:[3]

$$e_t = (m_t - m_t^*) + \eta(E_t e_{t+1} - e_t) - \phi(y_t - y_t^*) \qquad (3)$$

Solving Equation 3 forward, the dynamic exchange rate equation is given by.[4]

$$e_t = \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} E_t z_s \qquad (4)$$

where $z_t = (m_s - m_s^*) - \phi(y_s - y_s^*)$.

Assuming that the term z_t in Equation 4 follows an autoregressive process of order one, i.e. $z_t = \rho z_{t-1} + \varepsilon_t$, where $0 \leq \rho \leq 1$, and ε_t is white noise, Equation 4 implies that:

$$\begin{aligned} e_t &= \frac{1}{1+\eta}\sum_{\mathrm{s=t}}^{\infty}\left(\frac{\eta\rho}{1+\eta}\right)^{\mathrm{s-t}} z_t = \frac{1}{1+\eta-\eta\rho} z_t \\ &= \frac{1}{1+\eta-\eta\rho}\left[(m_s - m_s^*) - \phi(y_s - y_s^*)\right] \qquad (5) \end{aligned}$$

Equation 5 reflects the conventional model of the Monetary Approach to the Exchange Rate determination (M.A.ER).[5] Over recent years there has been a considerable effort to test empirically the predictions of various versions of the conventional M.A.ER model. Some recent works include Cushman (2000) who studied the Canadian/US dollar exchange rate

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[3] The model assumes identical coefficients η and ϕ across countries.

[4] Given the non-bubble solution that $\lim_{\mathrm{T}\to\infty}\left(\frac{\eta}{1+\eta}\right)^T E_t e_{t+T} = 0$

[5] Under appropriate rearrangements in Equation (3) the conventional M.A.ER model can also be expressed in terms of the nominal domestic exchange rate, and the nominal money supply, the nominal interest rate and the real output differentials between the domestic and the foreign economy, i.e. $e_t = (m_t - m_t^*) + \eta(\imath_{t+1} - \imath_{t+1}^*) - \phi(y_t - y_t^*)$.

reporting no evidence in favour of the monetary exchange rate model. Tawadros (2001) examined the Australian dollar/US dollar exchange rate and found a single long-run relationship among the exchange rate, money supplies, industrial output and interest rates. Crespo-Cuaresma et al., (2005) followed panel co-integration procedures in order to estimate the M.A.ER model for Central and Eastern European countries and reported long-run exchange rate relationships under the presence of the Balassa-Samuelson effect.[6]

The conventional M.A.ER model has also been augmented in order to examine empirically the effect of equities on the nominal exchange rate after introducing stock price effects. Under this specification the demand for money is assumed to depend on the level of interest rates, real income, and equity prices. Among others, Friedman (1988) and Boyle (1990) reported a significant relationship between stock prices and money demand. Caruso (2006), Hsing (2007), Cassola and Morana (2004) also supported the above relationship through multiple techniques such as co-integration testing.

Recently, Morley (2007) examined both an unrestricted and a restricted version of the conventional M.A.ER model after incorporating the real level of the stock market indices.[7] The basic modified M.A.ER model takes the form:

$$e_t = a_0 + (m_t - m_t^*) - a(y_t - y_t^*) + (\beta - 1)/\theta(i_t - i_t^*) - x(S_t - S_t^*) \qquad (6)$$

where i_t is the nominal rate of interest S_t is the log of the real level of stock market index and β, θ and x are parameters.[8] The unrestricted and restricted versions of Equation 6 are given respectively as:

[6] For an extended literature review behind the empirical validity of the conventional M.A.ER model see Wilson (2009).

[7] A market index was introduced following Friedman's (1988) specification.

[8] In Morley's model expectations are assumed to be formed regressively i.e. $E\Delta e_t = \theta(\bar{e} - e_t)$ where $\bar{e}$ the equilibrium exchange rate and θ a parameter that reflects the speed of adjustment. β is the interest rate coefficient and x the real stock market index coefficient (assumed the same for both the domestic and foreign country). All variables are expressed in logarithms except from interest rates.

Unrestricted model

$$e_t = \beta_0 + \beta_1(m_t) + \beta_2(m_t^*) + \beta_3(y_t) + \beta_4(y_t^*) + \beta_5(S_t) + \beta_6(S_t^*) + \beta_7(i_t) + \beta_8(i_t^*) + u_t \quad (7)$$

where u_t a random error term and the following restrictions are assumed to hold,

$$\beta_1 = -\beta_2;\ \beta_3 = -\beta_4;\ \beta_5 = -\beta_6;\ \beta_7 = -\beta_8$$

$$\beta_1, \beta_4 > 0;\ \beta_2,\ \beta_3 < 0;\ \beta_5, \beta_6, \beta_7 \beta_8 \lesseqgtr 0$$

Restricted Model

$$e_t = \lambda_0 + \lambda_1(m_t - m_t^*) + \lambda_2(y_t - y_t^*) + \lambda_3(S_t - S_t^*) + \lambda_4(i_t - i_t^*) + u_t \quad (8)$$

where $\lambda_1 > 0$, $\lambda_2 < 0$, $\lambda_3, \lambda_4 \lesseqgtr 0$

After estimating the model for the UK pound/US dollar exchange rate (using quarterly data for the period 1984 to 2002) Morley (2007) reported evidence that the model can produce a stable long-run relationship. Related to the effect of stock prices on the nominal exchange rate evidence was reported in favour of the unrestricted model (Equation 7) with a positive (although statistically insignificant) coefficient for the UK stock prices (S_t) and a negative and significant coefficient for the US stock prices (S_t^*).[9] Related to the restricted model (Equation 8), the evidence suggests a positive and significant effect of the stock price differential on the nominal exchange rate. Overall, evidence from both versions implies that the substitution effect (from money to equities) dominates the income or wealth effect, which suggests a negative relationship between the level of stock market prices and money demand.

In another study, using quarterly data for the period 1984 to 2004, Morley (2009) also compares the empirical validity of the conventional M.A.ER model (without stock price effects) with a model that incorporates equity effects (as reflected by Equation 8) reporting only limited evidence in favour of the conventional model for the UK/US exchange rate. In

[9] Under the augmented M.A.ER specification a positive coefficient for stock prices implies a depreciation of the nominal exchange rate and a negative coefficient implies an appreciation.

contrast, the alternative specification with equity prices produces evidence of a long-run relationship among the variables, with significant coefficients for the stock price differential, suggesting that the substitution effect dominates.

Following a similar methodology in order to investigate the long-run effect of stock prices on the exchange rate Sim and Chang (2008) have also employed Equation 8 and reported a stable long-run cointegration relationship among the variables tested (on a monthly basis from 1990 to 1997 and 1999 to 2008) for the Korean won *vis-à-vis* the U.S. dollar. The results are somewhat mixed, since for the pre-crisis period (1990 to 1997) evidence suggests a positive and insignificant coefficient for the stock price differential but a negative and significant coefficient for the post crisis period (1999 to 2008).

Following a different approach from the M.A.ER the portfolio balance model has also been employed in order to test empirically the effect of stock markets on exchange rates. Smith (1992) constructed a world model with optimal agent choice over risky assets in order to derive estimable exchange rate equations applied to German mark/US dollar and Japanese yen/US dollar exchange rates. Using data from US, Germany and Japan for the period 1974 to 1988 equities are shown to have a significant impact on the mark/dollar and yen/dollar exchange rates with a mixture of both positive and negative coefficients.

Given that the M.A.ER determination lacks fully articulated microfoundations, this paper contributes towards the portfolio balance approach by constructing a model with optimizing agents that incorporates an array of different assets within a two country world economy. As distinct from other research it is assumed that the optimizing agent is maximizing the present value of lifetime utility, given a sequence of budget constraints, where wealth is assumed to be allocated optimally among six different assets: domestic bonds; foreign bonds; domestic stocks; foreign stocks; domestic real money balances and foreign real money balances.

Explicit equations for domestic and foreign real money balances are derived, which can further be used to construct an equation that explicitly characterises the determination of the nominal exchange rate. The model specification is used in order to investigate for a potential long-run co-integration relationship among the UK pound sterling nominal effective exchange rate, the domestic and foreign nominal money balances, the domestic and foreign real output, the domestic and foreign stock prices, and the domestic and foreign interest rates.[10] Utility is assumed to be derived not only from domestic and foreign consumption but also from domestic and foreign real money balances. The predictions of the model are tested empirically, and when possible, a comparison with the predictions generated from the various versions of the augmented M.A.ER model, as previously analysed, is also attempted.

3. The Model

The infinitely lived representative agent (individual) is assumed to respond optimally to the economic environment. Utility is assumed to be derived from consumption of domestic and foreign goods and from holdings of domestic and foreign real money balances.[11] The presence of real money balances is intended to represent the role of money used in transactions, without addressing explicitly a formal transaction mechanism. This can distinguish money from other assets like interest bearing bonds or stocks.[12] The representative agent is assumed to maximize the present value of lifetime utility given by:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{({C_t}^{\alpha_1} {C_t^*}^{\alpha_2})^{1-\sigma}}{1-\sigma} + \frac{X}{1-\varepsilon} \left(\left[\frac{M_t}{P_t}\right]^{\eta_1} \left[\frac{M_t^*}{P_t^*}\right]^{\eta_2} \right)^{1-\varepsilon} \right] \tag{9}$$

[10] UK is assumed to be the domestic economy and USA a proxy for the foreign economy.

[11] Since the aim of this paper is to investigate the effect of stock prices on the nominal exchange rate and not to focus on potential effects of fiscal policy, debt deficit and debt management, the utility function employed does not incorporate government expenditure and risk associated with holding domestic money balances. For such specification see Kia (2006) and Wilson (2009).

[12] A direct way to model the role of money in facilitating transactions would be to develop a time-shopping model after introducing leisure in the utility function. Another approach, commonly found in the literature, allows money balances to finance certain types of purchases through a cash-in-advance (CIA) modeling. For tractability reasons the specification expressed by Equation (9) is adopted in this paper. See Walsh (2003) for the various approaches in modeling the role of money in the utility function.

where C_t and C_t^* are single, non-storable, real domestic and foreign consumption goods, $\frac{M_t}{P_t}$ and $\frac{M_t^*}{P_t^*}$ are domestic and foreign real money balances respectively, $0 < \beta < 1$ is the individual's subjective time discount factor, σ, ε, X are assumed to be positive parameters, with $0.5 < \sigma < 1$ and $0.5 < \varepsilon < 1$, and $E_t(\cdot)$ the mathematical conditional expectation at t. For analytical tractability, following Kia's (2006) suggestion, we assume that α_1, α_2, η_1, and η_2 are all normalized to unity.

The present value of lifetime utility is assumed to be maximized subject to a sequence of budget constraints given by:

$$y_t + \frac{M_{t-1}}{P_t} + \frac{M_{t-1}^*}{e_t P_t} + \frac{B_{t-1}^D(1+i_{t-1}^D)}{P_t} + \frac{B_{t-1}^F(1+i_{t-1}^F)}{e_t P_t} + \frac{S_{t-1}(P_t^S+d_{t-1})}{P_t} + \frac{S_{t-1}^*\left(P_t^{S,*}+d_{t-1}^*\right)}{e_t P_t} =$$
$$C_t + C_t^* q_t + \frac{M_t}{P_t} + \frac{M_t^*}{e_t P_t} + \frac{B_t^D}{P_t} + \frac{B_t^F}{e_t P_t} + \frac{S_t P_t^S}{P_t} + \frac{S_t^* P_t^{S,*}}{e_t P_t} \tag{10}$$

where y_t is current real income, $\frac{M_{t-1}}{P_t}$ and $\frac{M_{t-1}^*}{e_t P_t}$ are real money balances expressed in current domestic unit terms (with M_{t-1} and M_{t-1}^*domestic and foreign nominal money balances respectively carried forward from last period), e_t the nominal exchange rate defined as the amount of foreign currency per unit of domestic currency and P_t the domestic price index. B_{t-1}^D is the amount of domestic currency invested in domestic bonds at $t-1$ and i_{t-1}^D is the nominal rate of return on these domestic bonds. Similarly, $B_{t-1}^F$is the amount of foreign currency invested in foreign bonds at $t-1$ and i_{t-1}^F is the foreign rate of return on these foreign bonds. Both domestic and foreign bonds are assumed to be one period discount bonds paying off one unit of domestic currency next period. S_{t-1} and S_{t-1}^* denote the number of domestic and foreign shares respectively purchased at $t-1$, P_t^S and $P_t^{S,*}$ denote the domestic and the foreign share prices respectively and d_{t-1} and $d_{t-1}^*$the value of the domestic and foreign dividends earned.[13] q_t denotes the real exchange rate defined as $q_t = \frac{P_t^*}{e_t P_t}$ where P_t^* the foreign price index.

The agent is assumed to observe the total real wealth and then proceed with an optimal consumption and portfolio allocation plan. The right hand side in Equation 10 indicates that total real wealth is allocated at time t among real domestic and foreign consumption $(C_t, C_t^* q_t)$, real domestic and foreign money balances $(\frac{M_t}{P_t}, \frac{M_t^*}{e_t P_t})$, real domestic and foreign bond holdings $(\frac{B_t^D}{P_t}, \frac{B_t^F}{e_t P_t})$, and real domestic and foreign equity holdings $(\frac{S_t P_t^S}{P_t}, \frac{S_t^* P_t^{S,*}}{e_t P_t})$.[14]

The representative agent is assumed to maximize Equation 9 subject to Equation 10. In order to take an analytical solution for the intertemporal maximization problem, the Hamiltonian equation is constructed and the following necessary first order conditions (F.O.C) are derived:

$$\beta^t U_{c,t} - \lambda_t = 0 \tag{11}$$

$$\beta^t U_{c^*,t} - \lambda_t q_t = 0 \tag{12}$$

$$\beta^t U_{\frac{M}{P},t} \frac{1}{P_t} - \lambda_t \frac{1}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} \right] = 0 \tag{13}$$

$$\beta^t U_{\frac{M^*}{P^*},t} \frac{1}{P_t^*} - \lambda_t \frac{1}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} \right] = 0 \tag{14}$$

$$-\lambda_t \frac{1}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} (1 + i_t^D) \right] = 0 \tag{15}$$

$$-\lambda_t \frac{1}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} (1 + i_t^F) \right] = 0 \tag{16}$$

$$-\lambda_t \frac{P_t^S}{P_t} + E_t \left[\lambda_{t+1} \frac{1}{P_{t+1}} (P_{t+1}^S + d_t) \right] = 0 \tag{17}$$

$$-\lambda_t \frac{P_t^{S,*}}{e_t P_t} + E_t \left[\lambda_{t+1} \frac{1}{e_{t+1} P_{t+1}} \left(P_{t+1}^{S,*} + d_t^* \right) \right] = 0 \tag{18}$$

[13] It is assumed that the individual collects his dividend first and then goes out in the financial market to trade. In other words, the stock market opens after the realization of dividends.
[14] All variables are expressed in real domestic terms.

where λ_t the costate variable, $U_{c,t}$, $U_{c^*,t}$ the marginal utilities from domestic and foreign consumption and $U_{\frac{M}{P},t}$, $U_{\frac{M^*}{P^*},t}$ the marginal utilities from domestic and foreign real money balances respectively.

Dividing Equation 14 with Equation 16 and using Equation 12, Equation 19 below is obtained:

$$U_{\frac{M^*}{P^*},t} + U_{c^*,t}(1 + i_t^F)^{-1} = U_{c^*,t} \tag{19}$$

Equation 19 implies that the expected marginal benefit of holding additional foreign real money balances at t must equal the marginal utility from consuming foreign goods at time t. Note that the total marginal benefit of holding money at t is $U_{\frac{M^*}{P^*},t} + U_{c^*,t}$. Equation 19 can be rearranged in order to express the intratemporal marginal rate of substitution of foreign consumption for foreign real money balances as a function of the foreign bond return.

Dividing Equation 14 with Equation 18 and using Equation 12, Equation 20 below is obtained:[15]

$$U_{\frac{M^*}{P^*},t} + U_{c^*,t}\left[\frac{P_{t+1}^{S,*} + d_t^*}{P_t^{S,*}}\right]^{-1} = U_{c^*,t} \tag{20}$$

In a similar notion, Equation 20 implies that the expected marginal benefit of holding additional foreign real money balances at t must equal the marginal utility from consuming foreign goods at t. Equation 20 can be rearranged to express the intratemporal marginal rate of substitution of foreign consumption for foreign real money balances as a function of the foreign stock return.

Dividing Equation 13 with Equation 15 and using Equation 11, Equation 21 below is obtained:

[15] For notational simplicity we drop the mathematical conditional expectation $E_t(\cdot)$.

$$U_{\frac{M}{P},t} + U_{c,t}(1 + i_t^D)^{-1} = U_{c,t} \tag{21}$$

Equation 21 implies that the expected marginal benefit of holding additional domestic real money balances at t must equal the marginal utility from consuming domestic goods at t. Equation 21 can be rearranged to express the intratemporal marginal rate of substitution of domestic consumption for domestic real money balances as a function of the domestic bond return.

Finally, by dividing Equation 13 with Equation 17 and using Equation 11, Equation 22 below is obtained:

$$U_{\frac{M}{P},t} + U_{c,t}\left(\frac{P_{t+1}^S + d_t}{P_t^S}\right)^{-1} = U_{c,t} \tag{22}$$

In a similar notion, Equation 22 implies that the expected marginal benefit of holding additional domestic real money balances at t, must equal the marginal utility from consuming domestic goods at t. Equation 22 can be rearranged to express the intratemporal marginal rate of substitution of domestic consumption for domestic real money balances as a function of the domestic stock return.

Combining Equation 11 and Equation 12, Equation 23 can be derived:

$$\frac{U_{c,t}}{U_{c^*,t}} = \frac{1}{q_t} \tag{23}$$

Equation 23 implies that the marginal rate of substitution of foreign consumption goods for domestic consumption goods is equal to their relative prices.

Following Equation 9 the marginal utilities of consumption and real money balances can be derived as follows:

$$U_{c,t} = \beta^t (C_t)^{-\sigma} (C_t^*)^{1-\sigma} \tag{24}$$

$$U_{c^*,t} = \beta^t (C_t)^{1-\sigma} (C_t^*)^{-\sigma} \tag{25}$$

Dividing Equation 24 with Equation 25 and using Equation 23, Equation 26 is derived:

$$C_t^* = C_t(q_t)^{-1} \tag{26}$$

The marginal utilities for foreign and domestic real money balances are given respectively as:

$$U_{\frac{M^*}{P^*},t} = \beta^t X\left(\frac{M_t}{P_t}\right)^{1-\varepsilon}\left(\frac{M_t^*}{P_t^*}\right)^{-\varepsilon} \tag{27}$$

$$U_{\frac{M}{P},t} = \beta^t X\left(\frac{M_t^*}{P_t^*}\right)^{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{-\varepsilon} \tag{28}$$

Equations 19, 25, 26 and 27 imply that:

$$m_t^* = [(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t)^{\frac{1-\varepsilon}{\epsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}\right] \tag{29}$$

Equations 20, 25, 26 and 27 imply that:

$$m_t^* = [(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t)^{\frac{1-\varepsilon}{\epsilon}}\left[1-\left(\frac{P_{t+1}^{S,*}+d_t^*}{P_t^{S,*}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right] \tag{30}$$

Equations 21, 24, 26 and 28 imply that:

$$m_t = [(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\epsilon}}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\right] \tag{31}$$

Finally, equations 22, 24, 26 and 28 imply that:

$$m_t = \left[(C_t)^{1-2\sigma}(q_t)^{-[1-\sigma]}\right]^{-\frac{1}{\varepsilon}}\left[(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\epsilon}}\left[1-\left(\frac{P_{t+1}^{S}+d_t}{P_t^{S}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right] \tag{32}$$

Equations 29 to 32 reflect the demand equations for domestic and foreign real money balances (depicted by m_t and m_t^* respectively) as implied by the economic model. This system of equations can be used in order to solve explicitly for the determinants of the nominal exchange rate. Substituting Equation 30 in Equation 31 and Equation 32 in Equation 29, Equation 33 below is derived:[16]

$$le_t = \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 ly_t + \delta_4 ly_t^* + \delta_5 lP_t^S + \delta_6 lP_t^{S,*} + \delta_7 li_t^H + \delta_8 li_t^* \qquad (33)$$

where: $\delta_1 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_2 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_3 = -\left[\frac{1-\varepsilon}{\varepsilon}\right]$; $\delta_4 = \left[\frac{1-\varepsilon}{\varepsilon}\right]$; $\delta_5 = -\left[\frac{1-\varepsilon}{\varepsilon}\right]$;$\delta_6 = \left[\frac{1-\varepsilon}{\varepsilon}\right]$

$\delta_7 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_8 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]$

The predictions of the model are that:

$\delta_1 < 0$; $\delta_2 > 0$; $\delta_3 < 0$; $\delta_4 > 0$; $\delta_5 < 0$; $\delta_6 > 0$; $\delta_7 < 0$; $\delta_8 > 0$.

The following restrictions are assumed to hold:

$\delta_2 = -\delta_1$; $\delta_4 = -\delta_3$; $\delta_6 = -\delta_5$; $\delta_8 = -\delta_7$

$\delta_1 = \delta_7$; $\delta_2 = \delta_8$; $\delta_3 = \delta_5$; $\delta_4 = \delta_6$

4. Long-Run Empirical Methodology

In order to test empirically the validity of the economic predictions implied by Equation 33 in the long-run, a Vector Error Correction Model (V.E.C) is employed. The presence of a potential long-run co-integrating relationship among the variables in Equation 33 is investigated after normalizing with respect to the nominal exchange rate. In that sense the predictions of the economic model related to the determination of the nominal exchange rate in the long-run can be tested empirically. A comparison of the predictions of the model with the predictions from the augmented (M.A.E.R) model, as analysed in Section 2 (Equation 7), will be undertaken.

4.1 The vector error correction model; an empirical methodology

The vector autoregressive (VAR) approach, with normally distributed errors, is commonly used to describe and analyse the time series dependence of macroeconomic data. Under

[16] A l before a variable denotes log. See the Appendix for the full derivation of Equation 33 along with the various assumptions employed.

specific assumptions related to the normal time series vector process, this multivariate technique can be used for proper inference on macroeconomic relations.[17]

The traditional VAR process, which can be perceived as a reformulation of the covariances of the sample data, can be written as Equation 34 below:

$$\chi_t = \mu_0 + \Pi_1 \chi_{t-1} + \cdots + \Pi_k \chi_{t-k} + \varepsilon_t \qquad (34)$$

where χ_t a $(p \times 1)$ vector of variables, μ_0 an intercept term and ε_t a vector with the residuals. In order for a VAR to reflect rational economic expectations and further employed as a tool for economic experiments, the differences between mean and actual realizations in the model should be interpreted as white noise processes i.e. $\varepsilon_t \sim IN_p(0, \Omega)$ (Hendry and Richard 1983). The requirement for white noise residuals is crucial, not only for statistical inference but also for proper economic analysis. For this purpose, estimates should be considered as full information maximum likelihood (FIML) estimates.

In this paper an appropriate reformulation of the basic unrestricted VAR model is employed by introducing the so–called vector error correction model (VECM). Engle and Granger (1987) have shown that a set of integrated variables that are co-integrated should not only be estimated in first differences, because such an approach would ignore the long-run equilibrium relationships that can be found in time series data. As a result, they proposed that both short-run and long-run relationships of the variables should be accounted in a single econometric specification. This approach expresses the traditional VAR model in differences,

[17] Let $\{x_t\}$ be a stochastic process for $t = \ldots, -1, 0, 1, 2, \ldots$ If $E[x_t] = -\infty < \mu < \infty$ *for all* t, $E[x_t - \mu]^2 = \Sigma_0 < \infty$ *for all* t, $E[(x_t - \mu)(x_{t+h} - \mu)] = \Sigma_h < \infty$ *for all* t *and* $h = 1, 2, \ldots$ Then $\{x_t\}$ is said to be weakly stationary. Strict stationarity requires that the distribution of $(x_{t_1}, \ldots, x_{t_k})$ is the same as $(x_{t_{1+h}}, \ldots, x_{t_{k+h}})$ for $h = \ldots, -1, 0, 1, 2, \ldots$

lagged differences, and levels of the selected time series variables. As a result the VAR (k) model employed in this paper is of the following form:[18]

$$\Delta\chi_t = \Gamma_1^m \Delta\chi_{t-1} + \Gamma_2^m \Delta\chi_{t-2} + \cdots + \Gamma_{k-1}^m \Delta\chi_{t-k+1} + \Pi\chi_{t-m} + \varepsilon_t \quad (35)$$

where $\chi_t = \left(le_t, lM_t, lM_t^*, ly_t, ly_t^*, lP_t^S, lP_t^{S,*}, li_t^H, li_t^* \right)$ a $(9x1)$ vector of variables, m denotes the lag placement of the ECM term, Δ denotes the difference, and $\Pi = a\beta'$ with a and β (pxr) matrices with $r < p$, where p the number of variables and r the number of stationary co-integrated relationships.

To test for co-integration among a set of integrated variables the Full Information Maximum Likelihood (FIML) approach is employed as proposed by Johansen (1988, 1991).[19] Having uniquely identified potential co-integrating vectors, stationarity among the variables can be tested, while imposing specific restrictions. The above methodology is applied in the following section in order to test for a potential long-run relationship among the macroeconomic variables depicted in Equation 33.

5. Test for the long-run empirical validity of the economic modelling

Quarterly time series data for the United Kingdom and the USA are employed for the period 1982 to 2011for the variables depicted by Equation 33[20]. The beginning of the sample period was employed because in the early 1980's the UK fundamentally changed the definitions of its monetary aggregates ($M2$ definition of money supply in the UK now corresponds to $M1$ in

[18] Some of the advantages of this approach are that the VECM reduces the multicollinearity effect in time series, that the estimated coefficients can be classified into short run and long run effects, and that the long run relationships of the selected macroeconomic series are reflected in the level matrix Πthus can further be used for co-integration analysis. See Juselious (2006).

[19] The main advantage of such an approach is that it is asymptotically efficient since the estimates of the parameters of the short-run and long-run relationships are carried out in a single estimation process. In addition, through the FIML procedure potential co-integrating relationships can be derived in an empirical model with more than two variables.

[20] Data are collected from Datastream.

the USA) and both the UK and the USA deregulated their financial markets with important implications on the exchange rate and stock price relationship.[21]

Given the USA dollar's pivotal role in foreign exchange deals the USA is used as a proxy for the 'rest of the world', from the perspective of the UK. Within the UK's international environment, both in trade and in financial markets, the bilateral dollar/sterling exchange rate is unlikely to be a suitable empirical counterpart for the model's theoretical value. For this reason, the nominal effective exchange rate is employed that reflects a trade-weighted exchange rate for sterling. Given the dollar's relatively low weight in UK trade, the nominal effective exchange rate is more likely to represent the value of sterling abroad than the bilateral dollar/sterling exchange rate.

More specifically, le_t is the log of the UK nominal effective exchange rate, lM_t is the log of the UK nominal money supply ($M2$), lM_t^* is the log of the USA nominal money supply ($M1$), ly_t and ly_t^* are real GDP in the UK and the USA respectively, lP_t^S and $lP_t^{S,*}$ are the main stock market indices in the two countries, li_t^H is the log of $\frac{i_t^D}{1+i_t^D}$ where i_t^D is the three month rate on the UK Treasury securities and li_t^* is the log of $\frac{i_t^F}{1+i_t^F}$ where i_t^F is the three month USA Treasury bill rate.

In order to proceed with the VEC analysis the time series employed should be tested first for stationarity. Table 1 presents the results from the Augmented Dickey-Fuller (ADF) test under the null of a unit root. Evidence suggests (given the various levels of significance) that the first differences of the variables appear to be stationary as opposed to their levels.

[21] Data from the United States are used as a proxy for foreign variables. The UK and the USA were selected in the analysis as both economies have financial systems based on financial markets rather than on the banking sector as in most European economies. As shown in Morley (2002) this has an important implication for the relationship between stock prices and exchange rates.

Consequently, the variables can be considered to be integrated of order one, i.e. I (1), and cointegration among the variables is possible.[22]

Table 1. Augmented Dickey-Fuller test for a unit root.

	U.K.			
Variable	**Test in levels**		**Test in differences**	
	No Trend -	**Trend**	**No Trend -**	**Trend**
le_t	-2.11(0)	-2.24(0)	-9.44(0)*	-9.39(0)*
lM_t	-5.82(0)	2.79(0)	-8.42(0)*	-9.69(0)*
lM_t^*	-0.29(1)	-1.76(1)	-4.54(0)*	-4.48(0)*
ly_t	-1.79(1)	-1.23(1)	-5.07(0)*	-5.34(0)*
ly_t^*	-2.21(2)	-2.04(2)	-4.06(1)*	-4.52(1)*
lP_t^S	-2.44(1)	-1.91(1)	-8.25(0)*	-8.50(0)*
$lP_t^{S,*}$	-2.14(0)	-1.68(0)	-10.53(0)*	-10.66(0)*
li_t^H	-0.23(1)	-1.41(1)	-5.55(0)*	-5.76(0)*
li_t^*	2.71(1)	1.58(1)	-7.17(0)*	-7.72(00*

Note: Entries in parenthesis indicate the lag length based on SIC maxlag=12.
(*) indicates that the test is significant at all critical values.

Before testing for the co-integration rank, the appropriate lag length for the underlying empirical VECM model must be specified. Given the Langragian multiplier (LM) test for serial correlation of the residuals, 3 lags are employed for the model.[23] The Johansen (1995) procedures are then applied in order to test for the co-integration rank. The results from both the trace and the max-eigenvalue tests are given in Table 2. According to the tests, which indicate statistical evidence at 5% and 1% significant levels, three co-integrating vectors are employed.

[22] For robustness purposes we have also performed the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test with stationarity under the null. The KPSS also suggests that the variables are integrated of order one i.e. I(1).
[23] The AIC, SBA, HQ tests are employed for the lag order selection. Beginning with the lowest lag suggested by the tests (based on the SBC criterion) the serial correlation of the residuals is tested using the Langragian multiplier (LM) test.

Table 2. Johansen's co-integration rank tests.

	UK		
Hypothesized no of co-integrated relationships	**Trace Statistic**	**5% Critical Value**	**1% Critical Value**
None **	257.4334	192.89	204.95
At most 1 **	183.9152	156.00	168.36
At most 2 *	131.1575	124.24	133.57
At most 3	84.74999	94.15	103.18
At most 4	49.41512	68.52	76.07
At most 5	26.79050	47.21	54.46
At most 6	13.87705	29.68	35.65
At most 7	5.698096	15.41	20.04
At most 8	0.128072	3.76	6.65
Hypothesized no of co-integrated relationships	**Max-Eigen Statistic**	**5% Critical Value**	**1% Critical Value**
None **	73.51819	57.12	62.80
At most 1 *	52.75775	51.42	57.69
At most 2 *	46.40749	45.28	51.57
At most 3	35.33486	39.37	45.10
At most 4	22.62462	33.46	38.77
At most 5	12.91345	27.07	32.24
At most 6	8.178955	20.97	25.52
At most 7	5.570024	14.07	18.63
At most 8	0.128072	3.76	6.65

Note: *(**) denotes rejection of the hypothesis at the 5% (1%) level

As mentioned above, the rank of the Π-matrix was found to be $r = 3$ implying that statistically a discrimination among three conditionally independent stationary relations is possible. The three unrestricted co-integration relations are uniquely determined but the question remains on whether they can be meaningful for economic interpretation. Consequently, following Johansen and Juselius (1994), identifying restrictions should be imposed in order to distinguish among the vectors and ensure the uniqueness of the coefficients. By taking linear combination of the unrestricted β vectors, it is always possible to impose $r-1$ just identifying restrictions and one normalization on each vector without changing the likelihood function. Although the normalization process can be done arbitrarily it is generally accepted practice to normalize on a variable that is representative of a particular economic relationship. In that sense, since the purpose of the paper is to identify a possible long-run association between stock prices and nominal exchange rates (allowing always possible association of the nominal exchange rate with other variables) the first co-integrating vector is normalized with respect to the nominal exchange rate. Two additional restrictions (as implied by the economic model) are also imposed, namely that $\delta_1 = \delta_7$ and $\delta_3 = \delta_5$. The additional restrictions imposed on the other two vectors are depicted on the second and third column in Table 3 below.

In addition, all foreign variables, i.e. lM_t^*,ly_t^*,$lP_t^{S,*}$ and li_t^* , are treated as weakly exogenous variables, thus long run forcing in the co-integrating space. This is economically justifiable under the assumption that the UK is a small open economy, thus domestic policy decisions or more generally domestic economic activity do not have a significant impact on the evolution of foreign variables. Consequently, treating all variables as jointly endogenously determined would lead to inappropriate inference.

According to the Chi-squared value all restrictions are jointly accepted at four degrees of freedom. Consequently, the system is identified and according to Theorem 1 of Johansen and Juselius (1994) the rank condition is satisfied.

Table 3. Long-Run Co-integrating Relationships

le_t	**1.000000**	0.000000	0.000000
	-	-	-
lM_t	**-0.965236***	1.000000	0.000000
	[-7.00726]	-	-
lM_t^*	**1.427185***	0.000000	1.000000
	[10.0552]	-	-
ly_t	**-0.276830***	-0.821916	1.626847
	[-2.52576]	[-0.61346]	[1.18012]
ly_t^*	**1.434124***	-3.303591	-3.880012
	[4.34406]	[-2.25708]	[-2.57725]
lP_t^S	**-0.276830***	-0.512493	-0.430157
	[-2.52576]	[-1.97687]	[-1.62290]
$lP_t^{S,*}$	**0.077565****	0.858928	0.784978
	[0.72677]	[3.24828	[2.90528]
li_t^H	**-0.965236***	-1.458130	-2.134037
	[-7.00726]	[-3.71647]	[-5.29553]
li_t^*	**0.438966***	0.590043	0.977098
	[6.32874]	[3.06505]	[4.94383]
Constant	**-9.363495**	17.67103	-2.022253

Note: t statistics in brackets

(*) denotes that a coefficient is statistically significant at 5% level and correctly signed in accordance with the predictions of the model

(**) denotes that the coefficient is correctly signed in accordance with the predictions of the model but not statistically significant at 5% level

The first column in Table 3 reports the existence of a long-run co-integrating relationship normalized with respect to le_t. All variables appear to be statistically significant apart from the foreign price index. To test the stability of the VEC model the inverse roots of the characteristic AR polynomial are reported in Fig. 1. The analysis confirms that the VECM appears to be stable since the inverted roots of the model lie inside the unit circle, although a few roots are near unity in absolute value. Having established that the VEC model is stable the identified long-run co-integrating relationship, normalized on the nominal exchange rate, can be analyzed.

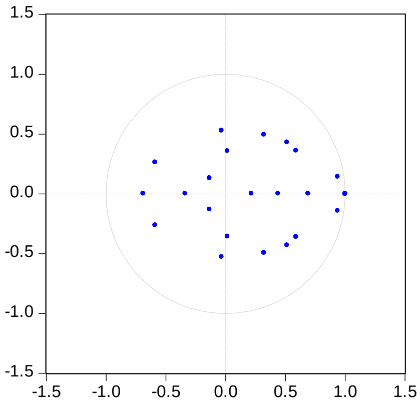

Fig. 1. Inverse roots of AR characteristic polynomial

5.1 Nominal money supply

The economic model (as reflected by Equation 33) predicts that an expansionary monetary policy in the UK results in a depreciation of the pound sterling i.e. $\delta_1 < 0$. The estimated coefficient for the domestic (UK) nominal money supply lM_t is negative, thus supporting the prediction of the model. The prediction also appears to be in line with the prediction of the M.A.ER model as reflected in Equation 7 in Section 2 i.e. $\beta_1 > 0$.[24]

The prediction of the constructed model related to an increase in the domestic money supply could reflect the fact that as the money supply increases the expectations of future depreciation are revised and therefore the pound depreciates. According to the M.A.ER model an increase in the domestic money supply will trigger an increase in the price level, which will lead to a depreciation of the domestic currency (given that the purchasing power parity holds in the long-run). Given that the uncovered interest rate parity also holds expectations for nominal future depreciation are also revised.

In a similar manner, the data supports the prediction of Equation 33 related to the foreign nominal money supply lM_t^* ($\delta_2 > 0$). The coefficient comes with a positive sign, implying that an expansionary monetary policy in the foreign economy (USA) will cause the pound sterling to appreciate as expectations of future pound sterling appreciation are revised. Again, this is evidence in favour of the prediction of the economic model (Equation 33), which also appears to be in line with the prediction of the augmented M.A.ER model ($\beta_2 < 0$ in Equation 7).

5.2 Real income

The economic model (as reflected by Equation 33) predicts that a higher real income in the UK will lead to a depreciation of the pound sterling i.e. $\delta_3 < 0$. The estimated coefficient for the domestic (UK) real income ly_t is negative. The evidence supports the prediction of the

[24] As opposed to the M.A.ER specification a negative coefficient in Equation 33 implies a depreciation of the currency.

economic model, which is consistent with a mechanism that links income to imports and thus to the exchange rate. The implication of such a mechanism is that higher income results in a higher demand for imports and thus leads to a depreciation of the domestic currency.

The results in the literature related to the way that domestic real income affects the nominal exchange rate over the long-run, are somewhat mixed. The M.A.ER model predicts an appreciation of the domestic currency on the grounds that an increase in the level of output in the long-run will induce a fall in the price level in order to produce an accommodating increase in real balances. Through the PPP the domestic currency then appreciates, while expectations about future appreciation are also revised. The prediction for a currency appreciation after an increase in domestic real income is also supported empirically by Morley (2007), after introducing stock price effects in the conventional M.A.ER model for the UK ($\beta_3 < 0$ in Equation 7), and by Wilson (2009) after constructing an optimization model without stock price effects for the USA. An explanation provided by Wilson (2009) is that a higher real income in the long-run may be perceived by foreign investors as evidence of future economic growth. This perceived growth may increase the belief that the domestic currency is a safe haven, which in turn causes a higher demand for the domestic currency thus leading to an appreciation of the currency.

This paper provides evidence of currency depreciation after an increase in domestic real income as predicted by the model constructed in Section 3, which introduces stock price effects under an optimizing framework. On similar grounds the coefficient for the foreign (USA) real income ly_t^* comes with a positive sign, which suggests (as the economic model predicts) that an increase in the foreign real income will lead to an appreciation of the pound sterling, $\delta_4 > 0$.

5.3 Share price indices

As previously discussed the model constructed in this paper has incorporated stock price effects within an optimizing economic framework in order to primarily investigate the effect of stock prices on the nominal exchange rate. The model predicts that as the UK share price index lP_t^S increases the currency (sterling pound) depreciates i.e. $\delta_5 < 0$ in Equation 33. According to Table 3 the estimated coefficient for the UK stock market index is negative, thus supporting the prediction of the model.[25]

A possible explanation is that as the price of equities increases, equities become more attractive to investors causing a substitution effect (which dominates the wealth or income effect) from money and other risk free assets towards equities. The demand of less risky assets relative to equities will decrease, implying a fall in their price and an increase in the interest rate. This increase in the interest rate will induce a further decrease in the demand for real balances. The price level will adjust in order to equilibrate the money market. Inflationary expectations will be revised upwards (given that the expected return on risky assets increases) which will further induce a current depreciation of the nominal exchange rate.

The substitution effect that leads to a depreciation of the nominal exchange rate in the long-run has also been reported by Morley (2007) when examining the unrestricted augmented version of the M.A.ER model ($\beta_5 > 0$ in Equation 7) where, surprisingly, the UK stock prices turned out to be insignificant. However, after employing Equation 8, Morley (2009) reports a positive and significant coefficient for a stock price differential for the country of the UK ($\lambda_3 > 0$).

[25] In this paper the UK stock price index is reflected by FTSE 100.

On similar grounds, in accordance with the prediction of the model ($\delta_6 > 0$), the coefficient for the USA stock price index $lP_t^{S,*}$ is positive, implying an appreciation of the pound sterling. The evidence is, however, weak as the coefficient appears to be statistically insignificant.

5.4 Interest rates

As the model predicts ($\delta_7 < 0$), the estimated coefficient for the domestic interest rate li_t^H is negative implying that as the domestic nominal interest rate increases the exchange rate depreciates. A possible explanation is that an increase in the domestic interest rate (possibly triggered by an unanticipated announcement for an increase in future growth rate of the money supply) will induce an adjustment in the price level in order to equilibrate the money market (given that expectations about future inflation are revised), and given the faster expected future depreciation of the UK pound against the US dollar, the exchange rate depreciates. A similar reasoning applies for the increase in the foreign interest rate, which induces a depreciation of the dollar and an appreciation of the UK pound, hence the positive sign for li_t^* in Table 3. The result is also consistent with the prediction of the economic model in Equation 33 that $\delta_8 > 0$.

6. Concluding remarks

This paper contributes towards the portfolio balance approach by constructing an intertemporal optimization model, which incorporates investment in an array of different assets, including domestic and foreign bonds, domestic and foreign stocks, and domestic and foreign real money balances, with a view to examine the effect of stock prices on the nominal exchange rate over the long-run. To date, such models incorporating the above asset choice have been neglected in studying the long-run relationship between nominal exchange rates and stock market prices. The predictions of the model were tested empirically using data from the UK and the USA for the period 1980 to 2011. On the whole the results tend to support the long-run validity of the economic model. Among other relationships, the model suggests that domestic stock market prices in the UK have a significant and negative association with the nominal exchange rate, after abstracting from short run fluctuations, suggesting that when UK stock prices increase, the pound sterling depreciates. Overall, the results are also broadly consistent with the predictions of the conventional monetary approach to the exchange rate (which however lacks fully articulated microfoundations) augmented with stock price effects. As with Morley (2007, 2009) the model provides evidence in favour of the substitution effect from money and other risk free assets towards equities.

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Appendix A: The derivation of the nominal exchange rate equation

Substituting Equation 30 in Equation 31 and Equation 32 in Equation 29 in the text the following equation is derived:

$$\frac{m_t}{m_t^*}=\frac{[(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}\left\{[(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}(m_t)^{\frac{1-\varepsilon}{\varepsilon}}\left[1-\left(\frac{P_{t+1}^{S,*}+d_t^*}{P_t^{S,*}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{[(C_t)^{1-2\sigma}(q_t)^{\sigma}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}\left\{[(C_t)^{1-2\sigma}(q_t)^{\sigma-1}]^{-\frac{1}{\varepsilon}}(X)^{\frac{1}{\varepsilon}}(m_t^*)^{\frac{1-\varepsilon}{\varepsilon}}\left[1-\left(\frac{P_{t+1}^{S}+d_t}{P_t^{S}}\right)^{-1}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}$$

which simplifies to:

$$\frac{m_t}{m_t^*}=\left(\frac{q_t{}^{\sigma-1}}{q_t{}^{\sigma}}\right)^{-\frac{1}{\varepsilon}}\left[\left(\frac{q_t{}^{\sigma}}{q_t{}^{\sigma-1}}\right)^{-\frac{1}{\varepsilon}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\left[\frac{m_t{}^{\frac{1-\varepsilon}{\varepsilon}}}{m_t^*{}^{\frac{1-\varepsilon}{\varepsilon}}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\frac{\left\{\left[\frac{P_t^{S,*}-\left[P_{t+1}^{S,*}+d_t^*\right]}{P_{t+1}^{S,*}+d_t^*}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}{\left\{\left[\frac{P_t^{S}-\left[P_{t+1}^{S}+d_t\right]}{P_{t+1}^{S}+d_t}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}\frac{\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}\qquad (A.1)$$

Dividing Equation 15 with Equation 17 yields that: $\frac{1}{P_t^S}=\frac{1+i_t^D}{P_{t+1}^S+d_t}$, which implies that:

$$P_t^S-[P_{t+1}^S+d_t]=-[P_{t+1}^S+d_t]\frac{i_t^D}{1+i_t^D}\qquad (A.2)$$

In a similar manner dividing Equation 16 with Equation 18 implies that:

$$P_t^{S,*}-\left[P_{t+1}^{S,*}+d_t^*\right]=-\left[P_{t+1}^{S,*}+d_t^*\right]\frac{i_t^F}{1+i_t^F}\qquad (A.3)$$

Using Equations A.2 and A.3, A.1 simplifies to:

$$\frac{m_t}{m_t^*}=[q_t{}^{\sigma-1}q_t{}^{-\sigma}]^{-\frac{1}{\varepsilon}}\left\{[q_t{}^{\sigma}q_t{}^{1-\sigma}]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[m_t{}^{\frac{1-\varepsilon}{\epsilon}}m_t^*{}^{-\left[\frac{1-\varepsilon}{\varepsilon}\right]}\right]^{\frac{1-\varepsilon}{\varepsilon}}\frac{\left\{\frac{\left[-\left[P_{t+1}^{S,*}+d_t^*\right]\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}{\left[P_{t+1}^{S,*}+d_t^*\right]^{-\frac{1}{\varepsilon}}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}{\left\{\frac{\left[-[P_{t+1}^{S}+d_t]\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{\left[P_{t+1}^{S}+d_t\right]^{-\frac{1}{\varepsilon}}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}}\frac{\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}}{\left[\frac{i_t^F}{1+i_t^F}\right]^{-\frac{1}{\varepsilon}}}$$

$$\frac{m_t}{m_t^*}=[q_t{}^{\sigma-1}q_t{}^{-\sigma}]^{-\frac{1}{\varepsilon}}\left\{[q_t{}^{\sigma}q_t{}^{1-\sigma}]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[m_t{}^{\frac{1-\varepsilon}{\epsilon}}m_t^*{}^{-\left[\frac{1-\varepsilon}{\epsilon}\right]}\right]^{\frac{1-\varepsilon}{\varepsilon}}$$

$$\left[P_{t+1}^{S,*}+d_t^*\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\frac{\left[P_{t+1}^S+d_t\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}{\left[P_{t+1}^{S,*}+d_t^*\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}\left[P_{t+1}^S+d_t\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{\frac{1}{\varepsilon}} \qquad (A.4)$$

Dividing Equation 17 with Equation 18 and using Equations 24, 25 and 26 implies that: $\frac{P_t^S}{P_t^{S,*}}=$

$\frac{e_{t+1}}{e_t}\frac{P_{t+1}^S+d_t}{P_{t+1}^{S,*}+d_t^*}$ which can be used to substitute for: $\frac{\left[P_{t+1}^S+d_t\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}{\left[P_{t+1}^{S,*}+d_t^*\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}}$ in equation A.4:

$$\frac{m_t}{m_t^*}=\left[{q_t}^{\sigma-1}{q_t}^{-\sigma}\right]^{-\frac{1}{\varepsilon}}\left\{\left[{q_t}^{\sigma}{q_t}^{1-\sigma}\right]^{-\frac{1}{\varepsilon}}\right\}^{\frac{1-\varepsilon}{\varepsilon}}\left[{m_t}^{\frac{1-\varepsilon}{\epsilon}}{m_t^*}^{-\left[\frac{1-\varepsilon}{\epsilon}\right]}\right]^{\frac{1-\varepsilon}{\varepsilon}}$$

$$\left[P_{t+1}^{S,*}+d_t^*\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^F}{1+i_t^F}\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_t}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^S}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_{t+1}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^{S,*}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}$$

$$\left[P_{t+1}^S+d_t\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}\left[\frac{i_t^D}{1+i_t^D}\right]^{-\frac{1}{\varepsilon}}\left[\frac{i_t^F}{1+i_t^F}\right]^{\frac{1}{\varepsilon}} \qquad (A.5)$$

which further implies that:

$$m_t{m_t^*}^{-1}={q_t}^{\left[\frac{2\varepsilon-1}{\varepsilon^2}\right]}{m_t}^{\left[\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]}{m_t^*}^{-\left[\frac{(1-\varepsilon)^2}{\varepsilon^2}\right]}\left[P_{t+1}^{S,*}+d_t^*\right]^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^*}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_t}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^S}^{-\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{e_{t+1}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{P_t^{S,*}}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}$$

$$\left[P_{t+1}^S+d_t\right]^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^H}^{\left[\frac{1-\varepsilon}{\varepsilon^2}\right]}{i_t^H}^{-\left[\frac{1}{\varepsilon}\right]}{i_t^*}^{\left[\frac{1}{\varepsilon}\right]} \qquad \text{(A.6)}$$

where $i_t^*=\frac{i_t^F}{1+i_t^F}$ and $i_t^H=\frac{i_t^D}{1+i_t^D}$

Taking logs of Equation A.6 yields:[26]

$$le_t=-\left[\frac{\varepsilon^2-(1-\varepsilon)^2}{1-\varepsilon}\right]lm_t+\left[\frac{\varepsilon^2-(1-\varepsilon)^2}{1-\varepsilon}\right]lm_t^*+\left[\frac{2\varepsilon-1}{1-\varepsilon}\right]lq_t-l\left[P_{t+1}^{S,*}+d_t^*\right]+\left[\frac{2\varepsilon-1}{1-\varepsilon}\right]li_t^*-lP_t^S$$

$$+le_{t+1}+lP_t^{S,*}+l\left[P_{t+1}^S+d_t\right]+\left[\frac{1-2\varepsilon}{1-\varepsilon}\right]li_t^H \qquad (A.7)$$

Using the fact that $m_t=\frac{M_t}{P_t}$, $m_t^*=\frac{M_t^*}{P_t^*}$ and $q_t=\frac{P_t^*}{e_tP_t}$ Equation A.7 becomes:

$$le_t=-\left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t+\left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t^*-\left[\frac{1-\varepsilon}{\varepsilon}\right]l\left[P_{t+1}^{S,*}+d_t^*\right]+\left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^{S,*}-\left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^S$$

$$+\left[\frac{1-\varepsilon}{\varepsilon}\right]l\left[P_{t+1}^S+d_t\right]+\left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^*-\left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^H$$

$$+\left[\frac{1-\varepsilon}{\varepsilon}\right]le_{t+1} \qquad (A.8)$$

[26] A l before a variable denotes log.

Following the fact that $\frac{P_t^S}{P_t^{S,*}} = \frac{e_{t+1}}{e_t}\frac{P_{t+1}^S+d_t}{P_{t+1}^{S,*}+d_t^*}$ and assuming that capital and consumption are homogeneous goods Equation A.8 becomes:

$$le_t = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t + \left[\frac{2\varepsilon-1}{\varepsilon}\right]lM_t^* + \left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^* - \left[\frac{2\varepsilon-1}{\varepsilon}\right]li_t^H - \left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^S + \left[\frac{1-\varepsilon}{\varepsilon}\right]lP_t^{S,*} - \left[\frac{1-\varepsilon}{\varepsilon}\right]q_t \qquad (A.9)$$

Substituting Equation 26 into Equation A.9 and following Kia's (2006) assumption that domestic and foreign real consumption (C_t, C_t^*) are a constant proportion ω of the domestic and foreign real income (where for simplicity it is assumed that $\omega = 1$) Equation A.10 is derived:

$$le_t = \delta_1 lM_t + \delta_2 lM_t^* + \delta_3 ly_t + \delta_4 ly_t^* + \delta_5 lP_t^S + \delta_6 lP_t^{S,*} + \delta_7 li_t^H + \delta_8 li_t^* \qquad (A.10)$$

where: $\delta_1 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_2 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_3 = -\left[\frac{1-\varepsilon}{\varepsilon}\right]$; $\delta_4 = \left[\frac{1-\varepsilon}{\varepsilon}\right]$; $\delta_5 - \left[\frac{1-\varepsilon}{\varepsilon}\right]$;$\delta_6 = \left[\frac{1-\varepsilon}{\varepsilon}\right]$

$\delta_7 = -\left[\frac{2\varepsilon-1}{\varepsilon}\right]$; $\delta_8 = \left[\frac{2\varepsilon-1}{\varepsilon}\right]$

Equation A.10 corresponds to Equation 33 in the text.

References

Boyle, G.W. (1990). Money demand and the stock market in a general equilibrium model with variable velocity. *Journal of Political Economy*, 98, 1039-53.

Cagan, P. (1956). The Monetary dynamics of hyperinflation, in *Studies in the Quantity Theory of Money*, F. Milton, University of Chicago Press, Chicago.

Caruso, M. (2006). Stock market fluctuations and money demand in Italy, 1913-2003. *Economic Notes*, 35, 1-47.

Cassola, N., & Morana, C. (2004). Monetary policy and the stock market in the euro area. *Journal of Policy Modeling*, 26, 387-99.

Crespo-Cuaresma, J., Fifrmuc, J., & MacDonald, R. (2005). The monetary approach to the exchange rates in the CEECS. *Economics in Transition*, 13, 395-416.

Cushman, D.O. (2000). The failure of the monetary exchange rate model for the Canadian-US dollar. *Canadian Journal of Economics*, 33, 591-603.

Engle, R. & Granger, C. (1987). Cointegration and error correction: representation, estimation and testing. *Econometrica*, 55, 251-76.

Friedman, M. (1988). Money and the stock market. *Journal of Political Economy*, 96, 221-45.

Granger, C., Huang, B.N. & Yang, C.W. (2000). A bivariate causality between stock prices and exchange rate: evidence from recent Asian flu. *Quarterly Review of Economics and Finance*, 40, 337-54.

Hendry, D. & Richard, J. (1983). The econometric analysis of economic time series. *International Statistical Review*, 51, 111-63.

Hsing, Y. (2007). Roles of the stock price and exchange rates in Slovakia's money demand function and policy implications. *Transition Studies Review*, 14, 274-82.

Johansen, S. (1988). Statistical analysis in cointegrated vectors. *Journal of Economic Dynamics and Control*, 12, 231-54.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica*, 52, 389-402.

Johansen, S. & Juselius, K. (1994). Identification of the long-run and short-run structure: an application to the ISLM model. *Journal of Econometrics*, 63, 7-36.

Johansen, S. (1995). *Likelihood–Based Inference in Cointegrated Vector Autoregressive Models*, A*dvanced Texts in Econometrics*, Oxford University Press, Oxford.

Juselious, K. (2006). *The Cointegrated VAR Model: Methodology and Applications,* Oxford University Press, Oxford.

Kia, A. (2006). Deficits, debt financing, monetary policy and inflation in developing countries: internal or external factors? evidence from Iran. *Journal of Asian Economics*, 17, 879-903.

Morley, B. (2002). Exchange rates and stock prices: implications for European convergence. *Journal of Policy Modeling*, 24, 523-26.

Morley, B. (2007). The Monetary model of the exchange rate and equities: an ARDL bound testing approach. *Applied Financial Economics*, 17, 391-97.

Morley, B. (2009). A comparison of two alternative approaches to exchange rate determination over the long-run. *International Econometric Review*, 1, 63-76.

Sim, S.H. & Chang, B.K. (2008). The monetary exchange rate model and stock prices: using the bounds testing approach. *Journal of Korea Trade,* 20, 1-24.

Smith, C.E. (1992). Stock markets and exchange rate: a multi-country approach. *Journal of Macroeconomics*, 4, 607-29.

Tawadros, G.B. (2001). The predictive power of the monetary model of exchange rate determination. *Applied Financial Economics*, 11, 279-86.

Walsh, C.E. (2003). *Monetary Theory and Policy*. (2nd ed.). Massachusetts Institute of Technology, Massachusetts.

Wilson, I. (2009). The monetary approach to the exchange rates: a brief overview and empirical investigation of debt, deficit, and debt management: evidence from the United States. *The Journal of Business Inquiry*, 8, 83-99.